

# New models of the unstable atmospheric surface layer

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## Abstract

For the last 50 years Monin-Obukhov similarity theory has provided a foundation for nearly all models of the atmospheric surface layer. The theory was developed when turbulence was still thought to consist of small parcels of fluid (eddies), with short lifetimes and erratic movements with respect to the mean flow. Such eddies would respond only to local conditions in the flow, so larger-scale influences were not considered. Recently two new models have been developed which assign much greater importance to the turbulence in the outer parts of convective boundary layers.

The new models of Hunt and Morrison (Euro. J. Mech. B - Fluids 19, 673-694, 2000) and McNaughton (Boundary-Layer Meteorol. 112, 199-221, 2004) both see the turbulent surface layer as being the direct product of eddies from the convective outer layer interacting with the ground. Hunt and Morrison propose that eddies from the outer layer impinge onto the surface where they are blocked by the surface and sheared by the mean wind, causing distortion. They use the linearizing assumptions of rapid distortion theory to analyze this process. By contrast, McNaughton proposes that the eddies from the outer layer create variable shear across the whole surface layer, which shear powers the development of 'TEAL' structures within the surface layer. These TEAL structures are attached to the ground, develop upwards by a cascade process, and are the fundamental elements of a self-organizing system of eddies within the surface layer. The TEAL model is incompatible with rapid distortion theory. The paper points out some testable predictions of the new models.

*Keywords: turbulence, Monin-Obukhov similarity, rapid distortion theory, self-organizing systems.*

## 1. Introduction

In the 1940s turbulence was understood in terms of Prandtl's mixing length model. This model saw transport by turbulence as analogous to transport by the molecules of a gas, but with whole parcels of fluid replacing the molecules. Though large compared to molecules, these 'eddies' were assumed to be very small compared to the outer dimensions of the flow. Like molecules, the eddies moved about erratically while being carried along by the mean flow. During their brief life they travelled a typical distance called the 'mixing length', analogous to the mean free path of gas molecules. Transport by such eddies was described by diffusion equations, so the problem of modelling turbulent transport became that of modelling the 'eddy diffusivity'. This depended on the mixing length and so only on the local properties of the flow.

Using these concepts, Obukhov (1946) assumed that buoyancy forces could affect transport only by changing the local properties of the flow: by changing the form (structure) and intensities (velocity scale) of the eddies. He represented the local influences by the friction velocity  $u_*$ , kinematic virtual heat flux  $w'\theta_v'$ , and the

buoyancy parameter  $g/\theta_v$ . Though not a local parameter, height,  $z$ , was added because it was known to be important when modelling the mixing length in boundary layers. Dimensional analysis based on these four parameters lead to Monin-Obukhov similarity theory.

This local view of turbulence is now known to be wrong. Eddies, or coherent structures, can be large, even spanning whole flows, and have dynamics that reflect non-local influences. For example, large convective eddies can span whole atmospheric boundary layers, so the variations in near-surface winds can depend on the depth of the whole boundary layer,  $z_i$ . Monin-Obukhov similarity theory, which neglects  $z_i$ , can be correct only if these large eddies do not affect vertical transport

processes near the ground. There is mounting evidence that this is not so, and that the outer layer must be considered when modelling transport in the surface layer (Högström, 1990; Khanna and Brasseur, 1997; Johansson et al., 2002; McNaughton and Brunet, 2002). The question now is how to understand this influence?

Here we compare two new models that address this question. These models agree on the importance of the outer turbulence in determining the properties of the surface layer, but they differ widely in their proposed dynamics.

## 2. The 'top-down' model

The top-down model proposes that turbulence in the atmospheric surface layer is created from eddies from the outer part of the boundary layer that impinge onto the ground. The model was proposed by Hunt and Morrison (2000), with further contributions by Hunt and Carlotti (2001), Högström et al. (2002) and Drobinski et al. (2004). This model is illustrated in Figure 1.

This model divides the surface layer into two parts: an upper part called the *shear* surface layer; and a lower part called the *eddy* surface layer. In the shear surface layer the eddies moving down from above are blocked by the ground while being sheared by the mean wind. Nearer the ground, in the eddy surface layer, these distorted large eddies scrape along the ground, creating intermittent internal boundary layers which then break down into substructures. The large-scale ramp structures observed by Hommema and Adrian (2003) are interpreted as this breakdown process in action. Turbulence in the surface shear layer achieves its distinctive form only in flows with Reynolds number high enough that velocity spectra display clear inertial subranges. A shear surface layer appears in the atmospheric boundary layer, but may be absent in laboratory flows with lower Reynolds numbers.

This conceptual model is analyzed using rapid distortion theory (RDT). The starting point is the frictionless Navier-Stokes equations

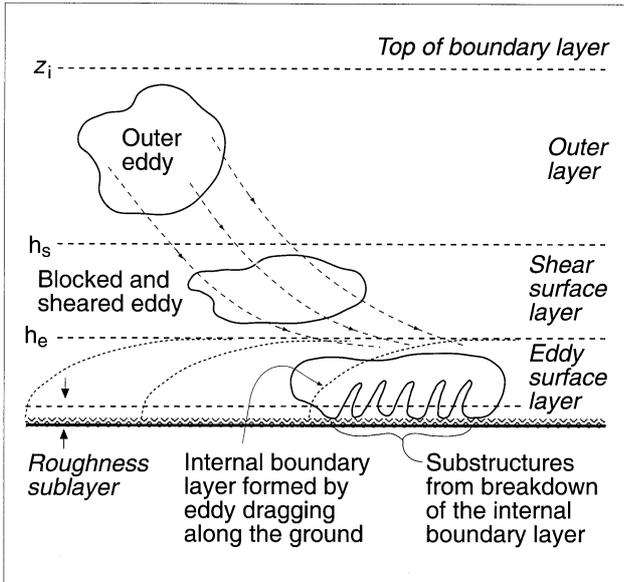


Figure 1. Schematic representation of turbulence processes in a convective boundary according to the Hunt 'top down' model. Here  $z_i \sim 1 - 2$  km,  $h_s \sim 100 - 200$  m,  $h_e \sim 10 - 20$  m. Adapted from Hunt and Carlotti (2001).

$$\frac{u_i}{t} = U_j \frac{u_i}{x_j} + u_j \frac{U_i}{x_j} + u_j \frac{u_i}{x_j} = - \frac{p}{x_i} \quad (1)$$

and the continuity equation

$$\frac{u_i}{x_i} = 0 \quad (2)$$

where symbols have their conventional meanings and the velocities have been split into mean,  $U_i$ , and fluctuating,  $u_i$ , components. These equations govern the dynamics of all eddies larger than a few millimetres. RDT linearizes (1) by neglecting the term before the '=' sign. This equation can describe the blocking and shearing of large eddies on time scales short compared to those of internal eddy processes, but it can not describe the transfer of energy from one scale of motion to another by non-linear processes, such as eddy break-up, amalgamation or growth. RDT is justified when eddies are 'rapidly' transformed by interaction with the surface, which is to say when the mean strain rate (the inverse of the time scale associated with the mean shear,  $S = (U_i/x)^{-1}$ ), is short compared to the evolutionary time scale of the eddies,  $s$ . It does not apply to eddies of the Richardson cascade, whose internal rate of strain is  $s = (k_1^2 \epsilon)^{1/3}$ . Here  $k_1$  is the streamwise wavenumber associated with the eddy. In the neutral surface layer ( $U_i/x \sim u_*^2/kz$  and  $\epsilon = u_*^3/kz$ , where  $k$  is von Kármán's constant. This means that RDT can apply only to eddies whose heights are large compared to height above ground.

Mathematical development of the top-down model is described in the cited papers. What concerns us here are the testable predictions arising from this analysis, especially the predictions of the forms of velocity spectra in the surface layers. These spectra behave differently in each of three wavenumber ranges. At the smallest

wavenumbers, up to  $k_1 \sim 1/s$ , the spectra reflect the motions of large eddies that fill the whole boundary layer and have horizontal lengths  $\sim s$ . They fill the boundary layer and so cannot move vertically or impinge onto the ground. Eddies of intermediate size can move about and can impinge onto the ground. The distortion of these eddies is analysed using RDT. Their final break-up occurs in the eddy shear layer. It is by non-linear processes that are beyond the scope of RDT. These eddies affect the spectra at wavenumbers  $k_1 > 1/z$ .

The general forms of various spectra predicted for the eddy shear layer are shown in Fig 3. The horizontal velocity spectra  $k_1 E_{11}(k_1)$  and  $k_1 E_{22}(k_1)$  have flat middle sections where  $E_{11}, E_{22} \sim k^{-1}$ .

### 3. The 'bottom-up' model

The bottom-up model proposes two kinds of turbulence whose interaction defines the top of the surface layer. The outer turbulence is created by buoyancy and its largest eddies span the whole boundary layer. In typical daytime conditions eddies from the outer layer do impinge onto the surface layer, where they are blocked and sheared, but the main effect of this is to transfer energy from the vertical wind components to the horizontal components at the top of the surface layer. This creates a variable horizontal wind at the top of the surface layer and so variable shear across the whole surface layer. This is superimposed on the mean shear and acts in exactly the same way as the mean shear. This unsteady shear drives production of turbulence within the surface layer which has entirely different characteristics.

In this model the turbulence within the surface layer is a self-organizing system of eddies that transfer momentum down to the ground. It is 'bottom-up' in the sense that instabilities form at the ground and initiate structures that grow upwards until they reach the top of the surface layer. These eddies draw their power from the local shear, so their transport properties depend on both the mean and the variable components of that shear.

The model is described by McNaughton (2004b, c). Its basic element is a coherent structure called the 'Theodorsen ejection amplifier-like', or TEAL structure. Each structure is initiated by an upward squirt of air, called an ejection. The oncoming flow lifts over, and curls around this creating a vortex with a hairpin-shaped core. This creates another, larger ejection from within its arc. A TEAL structure can thus initiate another, larger TEAL structure, so we can have TEAL cascades. Growing TEAL structures, compete for space by distorting each other. Only the best formed and most symmetrical at each stage produce ejections able to initiate a further cycle. The fragments of unsuccessful TEAL structures join the down-scale Richardson cascade to dissipation. TEAL cascades are 'inverse' cascades, in that they are in the inverse direction to the down-scale Richardson cascade.

The TEAL model predicts that the height of the layer dominated by TEAL structures is given by  $z_s \sim u_*^3/k \epsilon_o$ ,

where  $\epsilon_o$  is the dissipation rate in the outer layer. Here the upwards development of the largest TEAL cascades is checked by strong interactions with eddies of similar scale formed as part of the outer Richardson cascade. This defines the height of the surface layer. The layer

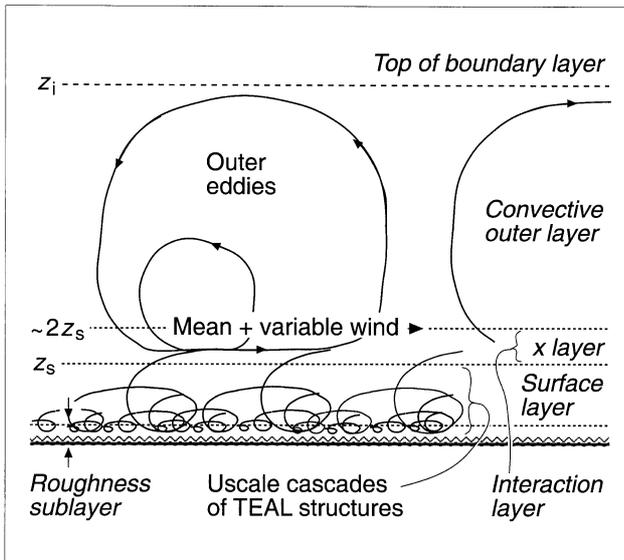


Figure 2. Schematic representation of turbulence processes in a convective boundary layer according to the TEAL model of McNaughton (2004b). Here  $z_i = 1 - 2$  km,  $h_s = |L| - 2|L|$ , where  $L$  is the Obukhov length.

where the interaction occurs (Fig. 2) has a thickness comparable to the size of the interacting structures, which is to say, it is about as thick as the surface layer itself. Buoyant production of turbulent kinetic energy roughly equals dissipation in the outer layer, so  $\epsilon_0$  depends on the surface heat flux and  $z_s$  becomes correlated with the Obukhov length,  $|L|$ . When  $z_i/|L|$  is large  $z_s$  is typically about one or two times  $|L|$  (McNaughton, 2004b). Thus the depth of the surface layer in the bottom-up model can vary from a few meters to the whole depth of the boundary layer. Typically it is 10 to 100m deep, so the 'surface layer' of the bottom-up model corresponds to the 'eddy surface layer' of the top-down model.

#### 4. Comparing models

The top-down and bottom-up models are based on quite different conceptual models of eddy processes in the surface layer. Theoretical arguments are incapable of deciding which, if either, is correct. Absolute theoretical proofs will remain beyond reach until the full Navier-Stokes equations can be solved. The RDT analysis of the top-down model is attractive if the non-linear term in (1) is indeed unimportant, but not otherwise. The TEAL cascade mechanism of the bottom-up model is attractive, but only if TEAL structures exist and behave as described in the model. The two models are also incompatible. The top down model proposes that the outer eddies penetrate the surface layer, break up and contribute their energy directly to the Richardson cascade in the surface layer. The TEAL model proposes that the outer eddies do not penetrate the surface layer but simply modulate the shear across it.

Despite these differences in concept we should not expect gross differences in model predictions because the construction of both models has been guided by experimental results. Even so, some of their predictions are different enough that meaningful comparisons can be made. In particular, the predicted forms of the horizontal velocity spectra in the (eddy) surface layer, are different

enough to be discriminated by careful experiments.

Fig. 3 shows  $k_1$  velocity spectra in the (eddy) surface layer as predicted by the two models. The predicted spectra for vertical velocity,  $E_{33}$ , are the same, but predictions for the horizontal spectra are significantly different. The top-down model predicts spectra that stretch continuously from the smallest wavenumbers to the largest ones, and predicts that  $E_{11}, E_{22} \sim k_1^{-1}$ . That is to say, the spectra plotted in Fig. 3 as  $k_1 E_{11}$  and  $k_1 E_{22}$  have flat mid sections. This is a central prediction of the top-down model.

By contrast, the bottom-up model predicts that  $k_1 E_{11}$  spectra have two parts—an 'inner' part at larger wavenumbers reflecting the TEAL structures and their break-down products, and an 'outer' part at smaller wavenumbers representing the modulation of the inner turbulence by the outer eddies. These spectral parts simply overlap, and the heights and frequencies of the outer and inner peaks are governed by different parameters. These predictions are also illustrated in Fig. 3.

In Fig. 3 the heights of the two peaks of the bottom-up  $k_1 E_{11}$  spectra are drawn with equal heights. This is often observed in the surface layer, and it makes flat the linking spectrum between two peaks. This flat section is indistinguishable from the  $k^{-1}$  spectrum predicted by the top-down model. Such observations do not discriminate between the two models. A stronger test is to compare slopes of  $k_1 E_{11}$  spectra at several heights. The top-down model predicts mid regions of  $k_1 E_{11}$  spectra to be flat at all heights within the surface layer, while the bottom-up model predicts the slope of this linking region to decrease progressively with height, moving towards negative values as the outer peak grows relative to the inner peak. Lauren et al. (1999) find slopes to increase

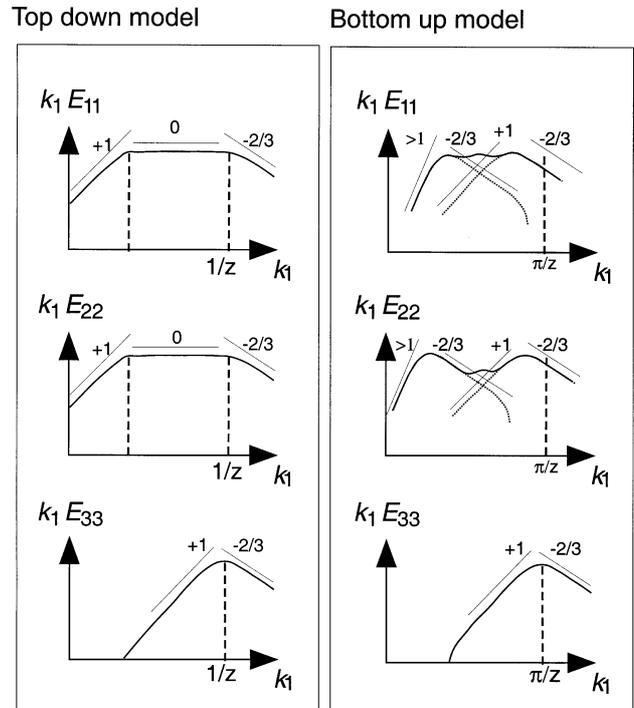


Figure 3. Spectra of wind components in the atmospheric eddy surface layer according to the top-down and bottom-up models. Axes are logarithmic.

with height, but this single result needs confirmation by results from a more extensive site.

Transverse velocity spectra,  $k_1E_{22}$ , can also be compared. The few such spectra published show outer and inner peaks to be spread more widely than in  $k_1E_{11}$  spectra. The inner peak is at a wavenumber about twice that in the  $k_1E_{11}$  spectrum. This has been ascribed to the different effects that streamwise aggregation of attached eddies has on the  $k_1E_{22}$  and  $k_1E_{11}$  spectra (McNaughton, 2004a). Observations show the outer peak of  $k_1E_{22}$  to be typically at about half the wavenumber of the corresponding outer  $k_1E_{11}$  peak (Nicholls and Readings, 1981; McNaughton and Brunet, 2000; Hong et al., 2004). This, presumably, is because the outer convective cells are partly aligned with the mean wind. The predictions of the top-down model are not sensitive to the separation of the shoulders of these spectra, so it again predicts a flat mid spectrum. The bottom-up model, on the other hand, predicts that a spectral gap will open up to reveal the separate natures of the two peaks. This gap would be larger when peak separation is maximized by taking observations near to the ground within a deep boundary layer. The few published results do show a gap, but confirmation is needed.

Cross-wind spectra can be observed by aircraft flying across the mean wind, and these can also be compared with model predictions. Unfortunately  $k_2E_{11}(k_2)$ , and  $k_2E_{22}(k_2)$  spectra are rarely published, but those that are show the outer and inner spectral peaks to be spread more widely in these than in spectra taken along the wind. The wavenumber of the inner peak of the  $k_2E_{11}(k_2)$  spectrum is at a wavenumber about a decade larger than in the along-stream spectrum  $k_1E_{11}(k_1)$ . The explanation given by the bottom-up model is that the streamwise alignment of attached eddies in the surface layer does not affect  $k_2$  spectra (McNaughton, 2004a). The  $k_2E_{11}(k_2)$  spectrum of Nicholls and Readings (1981) shows increased separation and a distinct spectral gap in the  $k_2E_{11}(k_2)$  spectrum. This is a single result that requires confirmation.

## 5. Conclusion

Two new models have been proposed to explain how turbulence processes in the outer part of the boundary layer can affect turbulent transport near the ground. Both models introduce outer-layer parameters, so neither is compatible with Monin-Obukhov similarity theory. However, the two models are also incompatible with each other.

This brief paper has outlined the nature of the two models and their incompatibility. It also points out that the two models can be discriminated experimentally by comparing the turbulent spectra they predict with experimental observations. The most useful observations are also those least reported: transverse velocity spectra measured from an aircraft flying along the wind, or from a stationary tower using Taylor's frozen turbulence hypothesis,  $k_1E_{22}(k_1)$ ; and cross-wind spectra of horizontal velocity components which can be observed from aircraft,  $k_2E_{11}(k_2)$  and  $k_2E_{22}(k_2)$ . The top-down model of Hunt and Morrison (2000) and followers predicts these spectra to be continuous with flat middle sections, while the 'bottom-up' model of McNaughton (2004b) predicts that separate peaks with a spectral gap

between will be apparent when the two peaks are widely separated. For optimal discrimination spectra should be measured at low heights within a deep convective boundary layer.

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